DESIGNING THE WORLD AROUND ME: MATHEMATICS AND CULTURAL INSPIRATION IN DESIGN

Description

The activity supports the development of geometric understanding through the use of two dimensional shapes in cultural and religious symbolism and though their use in architecture, particularly in the use of tiling. Throughout there are opportunities for exploring how symbolic representations are underpinned by ideas and beliefs. There are seven tasks.

Global citizenship competences addressed

- appreciate different perspectives & world views
- positive interactions with people who are different
- take constructive action for social well-being
- analytical & critical thinking skills
- communication & co-operation skills

Global citizenship content

Inclusive social relationships; intercultural exchanges; knowledge of other cultures; historical awareness

Mathematical approaches

- looking for patterns and connections
- asking yourself questions
- visualising, imagining and using intuition
- using argumentation and reasoning
- recognising the social and ethical dimensions of mathematics
- questioning the use of mathematics in structuring experience of the world

Mathematical content

Names and properties of two dimensional shapes; reflective and rotational symmetry, factors, primes ad co-primes; regular and semi-regular tiling

Resources required

Rulers; 15, 20, 25, 30 and 48 point circles; colours and a variety of design resources; access to computers; a plentiful supply of triangles, squares and hexagons with equal side length; cameras

Time needed (in and out of the classroom)

Approximately eight hours curriculum time.

Organization and practical issues

Whole class teaching plus small groups. Task 6 is best begun as a homework activity.

Suggested plan for teaching

The activity is built around 6 tasks, plus an optional one. Overall intended learning outcomes could include the ability to:

- recognise two dimensional shapes
- classify shapes using various criteria
- provide examples of the use of different shapes for decoration
- identify and construct regular and semi-regular tiling patterns
- see patterns involving factors, primes and co-primes
- identify the main universal symbols (such as religious symbols)
- put in relation geometric shapes, their social (attributed) meaning and their use in decorating significant institutions
- use shapes and patterns in order to represent (social) relations

Task 1: Designing with regular polygons (approximately 1½ hours)

Use PowerPoint slides 1, 2, 3, 4, and 5 as a stimulus.

Image credits: User:Vmenkov [CC BY-SA 3.0 (https://creativecommons.org/licenses/by-sa/3.0)] https://commons.wikimedia.org/wiki/File:Wuhan_-_Future_City_-_Area_G_-_P1530366.JPG

Imagine you are designers of the future.

Explain that this activity is about using mathematics and our cultural heritage to create pleasing designs.

What does a designer need to know in order to make people feel good?

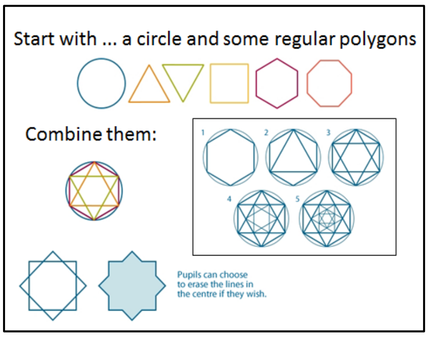
Possibilities are mathematics, aesthetics, imagination, creativity, knowledge, skills, a committed attitude, practice.

Slide 6 sets up the task.

What is a polygon?

It is a closed 2-D shape with straight edges. Give counter examples to show why each part of the definition is needed.

What is a regular polygon?



Images credits: School of Islamic Geometric Design (http://www.sigd.org/resources/)

A polygon with all sides and all angles equal. Ask the students to sketch shapes that show, again, that both parts of the definition are needed.

Give children 48 point circles and ask them to make a design using the shapes from slide 6 and/or any other regular polygons that can be made by joining the dots.

Use PowerPoint slides 7, 8, 9, 10, 11 and 12 to discuss reflective and rotational symmetry only.

Which have rotational symmetry only?

Use the vocabulary of orders of symmetry. (All symbols have rotational symmetry order 1 but we do not say these shapes have rotational symmetry.)

In small groups students discuss the symmetries of the designs they have produced. Almost all of these will have reflective and rotational symmetry. Each group shares one of their designs with the class and its symmetries are discussed.

Draw out that any shape with reflective symmetry order 2 also has rotational symmetry order 2.

Make two copies of your design. Colour one so that it has reflective and rotational symmetry and the other so that it has rotational symmetry only.

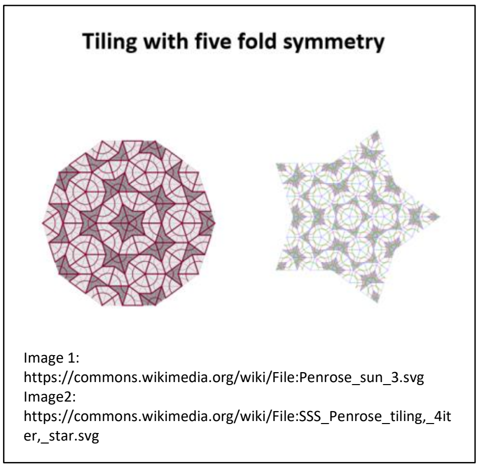
☺+☺ Make a display and share the results through e-twinning.

Task 2: Pentagonal symmetries (1 hour)

What orders of symmetry did our designs have? What do you notice about all these numbers?

They are factors of 48. What orders could we not make on a 48 point circle?

Use PowerPoint slide 13 to discuss the orders of symmetry that are found in Islamic designs. Slide 14 shows some images of Islamic tiling with five fold symmetry.



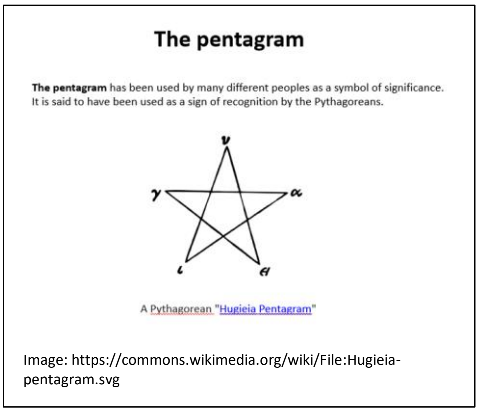
Discuss the pentagon and the five pointed star - the pentagram. The pentagram has been important in many different cultures. It is said to be the secret sign of the Pythagoreans.

Ask ten students to stand in a circle. They throw a ball of wool one space round the circle.

What shape have we made? How many spaces round the circle would make a pentagon?

Ask a second group of ten to show this.

How many spaces round the circle would create a pentagram?



They discuss this in small groups and then a third group of ten try out different people's ideas.

Hand out 15, 20, 25 and 30 point circles to different groups.

How many spaces to create a pentagram on your circle?

Any who finish quickly can think about a 35 point circle. Collect all the results.

What do you notice about these numbers?

Use the words ratio and fraction in supporting the students' discussion.

Using the electronic worksheet http://tube.geogebra.org/material/show/id/1385121 the students test out their predictions for 5, 35 and 40.

Task 3: Exploring patterns in point circles (1 hour or longer if explore in greater depth)

Exploring point circle patterns further allows rich opportunities to think about factors, prime and co-prime numbers.

Encourage the students to ask as many of their own questions as possible. There are many different questions that can be asked. Examples include:

What happens with a jump size 1?

Can I always make the same shape a different way on the same circle?

When can I make a square? A triangle? A pentagon? And so on.

When do I visit every point?

Are there some circles where I always visit every point?

What sort of numbers are these?

If I count the number of lines and the number of times we go round the circle, can we make any predictions?

Ideally the students will have access to the electronic worksheet. Otherwise a good variety of point circles will need to be available.

Task 4: Designing a symbol to represent our class (1 hour)

Slides 6 to 12 include a variety of symbols from a variety of different cultures and historical times. All of the symbols have mathematical properties. Discuss the symbols with the class and encourage the students to discuss the symmetries of the symbols and the meanings conveyed by this and by other properties of the designs.

Using everything they have learnt so far, including the meanings of symbolic representations, the students work in small groups to design a symbol to represent their class. Begin with a discussion about what is important about the values that the class share, what types of relationship they might depict and so on.

What motifs might you use? How will you represent our relationships and our learning community? How will you use symmetry?

Follow this with a discussion about the designs. What types of relationships have they depicted? What shapes did they use and why? Did any of them use symmetry to represent relationships? Did any of the groups try to represent fairness and balance?

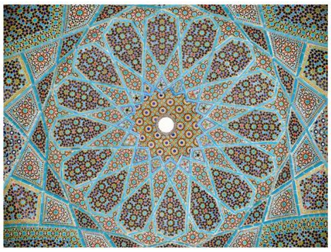
☺+☺ The class may wish to share their symbols through e-twinning.

Task 5: Exploring tiling mathematically (2 hours)

Use slide 16 and the 3 minute movie about Isfahan (https://www.youtube.com/watch?v=QqbiDdsaZw4) as a stimulus to think about tiling patterns.

Share this simplified version of an article and the three images from the New Scientist (https://www.newscientist.com/article/dn11235-medieval-islamic-tiling-reveals-mathematical-savvy/):

Medieval Islamic tiling reveals mathematical savvy



Medieval Islamic designers used elaborate geometrical tiling patterns at least 500 years before Western mathematicians developed the concept.

The geometric design, called girih, was widely used to decorate Islamic buildings but the advanced mathematical concept within the patterns was not recognised, until now. The 15th-century tiles formed so-called Penrose geometric patterns. Penrose tiling is a concept developed in the West only in the 1970s.

Girih designs were assembled from five regularly shaped tiles, including a bowtie shape, a rhombus, a pentagon, an elongated hexagon, and a decagon (slide 17).

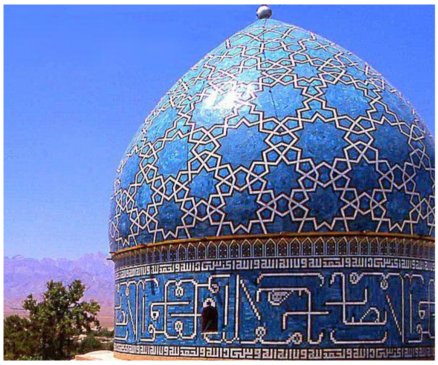


Image source: https://commons.wikimedia.org/wiki/File:Shah_Nematollah_Vali_Shrine_13_detail.jpg Author: anaareh saaveh

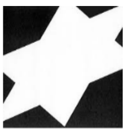
The atoms in certain materials can arrange themselves in similar non-repetitive patterns, which are called quasi-crystals. They are called this because they have a well-defined structure but the atoms are not spaced uniformly as in a normal crystal.

The correspondence between mathematically determined design and the natural world is always intriguing. The students may like to consider if there are other types of correspondences – between mathematics and soul, for instance.

Explain that the students are going to begin by thinking about very simple, regularly repeating tiling patterns.

The simplest tilings use just one shape. If we use just regular polygons we make a regular tiling. How many are there? How do we know we have found them all?

To explore further, you will need a good supply of triangles, squares and hexagons sharing the same side length. A good supplier of these can be found at https://www.atm.org.uk/Shop/MATs---View-All

First the students freely explore tiling patterns using some or all of these three shapes. Introduce the rule that the tiling patterns must be *periodic*, that is, the pattern repeats regularly across the plane in all directions. They take photographs of designs they find pleasing.

If we introduce a new rule, we will find that only a few of these patterns are allowed.

A semi-regular tiling uses regular polygons of more than one type. In addition, every vertex where the tiles meet has to be the same.

Are any of our patterns made so far semi-regular?

There are five semi-regular tilings that can be made with these shapes. Can you find them all?

Can you prove there aren't any more? (Hint: Consider the possible ways the shapes can meet at a vertex.)

Task 6: Tiling patterns in the world around me (1 ½ hour)

Show the students an image of one of the semi-regular tilings from the last activity found in the tile floor from the Museo Arqueológico de Sevilla, Spain (slide 18) (https://commons.wikimedia.org/wiki/File:Semi-regular-floor-3464.JPG)

Tiling patterns are used all around us in both ancient and modern buildings.

The students look for tiling patterns in the environment. They take photographs to share and discuss the mathematical properties of the tilings they have found.

Task 7 (optional): Reflection on the nature/ rules of the social world around me (1 hour)

In Task 4 we looked at the Share symbol. You designed a mathematical symbol to represent the class. And you thought a lot about symmetry.

Image credit: Background vector created by freepik - www.freepik.com (cropped)

Ask students to think if the social relationships could be represented geometrically and what would be the rules. (Slide 19)

There are number of approaches for the discussion. For example (Slide 20):

(1) Similarities & differences.

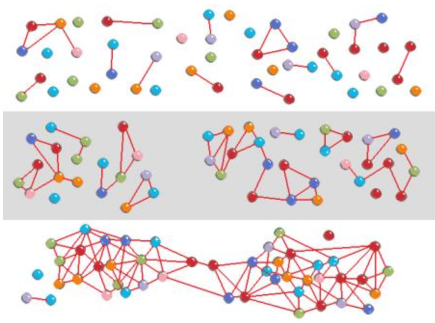


Image source: https://en.wikipedia.org/wiki/File:Network_self-organization_stages.png Author: Takemori39

Think of the points or circles you use to represent each of you. Do they have the same (colour and) size? Why not? Doesn't this add some beauty to the world?

Think of the lines connecting you in pairs. Are they equal? Why not?

What is putting us close and what is setting us apart?

(2) Present & future. Permanent & changing. The evolving nature of human relationships.

It is likely that the groups of friends you are in will change over time – class members with whom you barely speak now can become your best friends. What is then the value of the drawing you would do now to represent the relationships in your class? Could you ever do a tiling for this?

Creating connections. *Think of how you first met your best friend. Were you friends from the beginning or you become friends over time?*

The world is a moving tile. *What would be necessary to become friend with somebody from the other part of the world?*

(3) Subjective versus objective.

If an external person who didn't know you at all represented your class, would the drawing look different? Would the circles representing each pupil be the same? What about the lines connecting you – would they have the same length? What does this mean?

Do you think you could find something in common with every child on this planet?

Extending the learning

Possible discussion themes to be explored further starting from the students' drawings attempting to represent their class and their discussion about picturing social relations are:

- equality
- diversity
- inclusion
- equity

They should probably be approached in-depth in a civic education class or in a dedicated non-formal activity involving all class members.

Other resources (material and human resources)

Videos:

http://www.etereaestudios.com/docs_html/isfahan_htm/isfahan_movie_index.htm#

http://www.etereaestudios.com/docs_html/arsqubica_htm/index.htm

Web links:

School of Islamic Geometric Design. Resources. Online: http://www.sigd.org/resources/

School of Islamic Geometric Design. Basic Design Principles. Online: http://www.sigd.org/resources/basic-design-principles/

Lu, Peter J. & Steinhardt, Paul J. (2007). Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture. In: *Science* 23 Feb 2007: Vol. 315, Issue 5815, pp. 1106-1110. DOI: 10.1126/science.1135491 Online: http://science.sciencemag.org/content/315/5815/1106

Dunham, Will (2007). Islamic maths was 500 years ahead. [Reuters] Online: http://www.abc.net.au/science/articles/2007/02/23/1855313.htm

Hecht, Jeff (2007). Medieval Islamic tiling reveals mathematical savvy. In: Daily News. Online: https://www.newscientist.com/article/dn11235-medieval-islamic-tiling-reveals-mathematical-savvy/

National Centre for Excellence in the Teaching of Mathematics (2009). The Art of Mathematics Islamic patterns. In: Primary Magazine - Issue 13: The Art of Mathematics. Online: https://www.ncetm.org.uk/resources/18030

NRICH Enriching Mathematics. Islamic Tiling. Tiling with Equilateral Triangles. Serendipity. Online: https://nrich.maths.org/1561 , https://nrich.maths.org/1545 , https://nrich.maths.org/1559

Images:

https://www.ancient-symbols.com/religious_symbols.html

http://mathworld.wolfram.com/HeartCurve.html

https://www.shutterstock.com/search/social+science

Ethical issues or dilemmas

Designing a symbol to represent the class may raise ethical dilemmas especially with respect to inclusion and diversity:

We have similar minds and hearts - why don't we have the same beliefs and expectations? Do we all have the same rapport one to each other? Based on what criteria do you share ideas or resources with your classmates? Can the relationships among members of your class can be represented regularly, as equidistant from one to another?

Are differences among people beneficial or harmful for a group? Do we have the same type of shrine? How do other beliefs guide the life of people?

In Task 4, guided by teacher, students are exploring and discussing significations attributed to different religious symbols. The teacher should avoid attempts to categorise or assign inappropriate connotations to symbols or to ideological conceptions.