Project in Citizenship and Mathematics
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Exploring Growth

How fast do rabbits reproduce?

Description

The pupils are studying the growth of the rabbit population in Australia and its effects on nature and humans. They get to know the topics growth and correspondence and work out suitable models and representations. They playfully conduct a conflict resolution of this global challenge.

The activity is divided into 5 tasks: *1. Picture story "How the rabbits came to Australia", 2. Number of rabbits - estimation and approximation, 3. Growth of the rabbit population, 4. Preparation of the role playing game, 5. Role playing game*

Global citizenship competences addressed

- Capacity to examine global issues
- Appreciate different perspectives & world views
- Positive interactions with people who are different
- Take constructive action for sustainable development & social well-being
- Analytical & critical thinking skills
- Communication & co-operation skills
- Conflict resolution skills

Global citizenship content

Environmental Sustainability; Legacy of Colonialism; Consider the world as a whole; Interdependence; Invasive species; Recognize and analyse global challenges; Solving conflicts and working out compromises

Mathematical approaches

- Looking for patterns and connections
- Being organised and systematic
- asking yourself questions
- Being resilient and flexible
- Conjecturing and checking things out
- Modelling and dealing with uncertainty
- Questioning the use of mathematics structuring experience of the world
- Recognising the political and ethical dimensions of mathematics
- Using argumentation and reasoning
- Using representation and symbolism

Mathematical content

Estimations; Approximations; Correspondence; Growth; Big numbers; Create and critically reflect modelling and representations

Resources required

For the class: Picture Story "How the Rabbits came to Australia" (text for reading aloud and illustrations printed in Din A4 or Din A3, preferably on solid paper), Globe or big world map, Slides of the PowerPoint presentation, Sound bowl or bell

For every student: printed text "Rabbits in Australia"

For each small group of 2 or 3 students: printed sheet "Many Rabbits", printed sheet with 35 rabbit cards

For each discussion group: printed role cards (rabbit, farmer, teacher, observation, optional animal rights activist)

Time needed (in and out of the classroom)

About four lessons (two double lessons)

Organization and practical issues

The students work in class, in groups of 2 or 3 and later in discussion groups consisting of 8 - 0 students. The activity begins in a chair circle, for the group work group tables are needed.

Suggested plan for teaching

Task 1: *Picture Story "How the Rabbits came to Australia" (15 minutes)*

A picture story introduces the lesson. The students sit in a circle. The printed pictures are distributed to individual students. They will hold them up when they are asked to do so and it's their picture's turn. The teacher reads the story. The individual students are asked at the corresponding point in the story to hold up the pictures so that everyone can see them.

Afterwards, the students talk about the story and their impressions. The teacher can moderate the conversation, for example by using the following questions:

Who has ever seen a rabbit?
Who has a rabbit at home?
Where do rabbits live? Do rabbits live here as well?
Does anyone know a hunter?
Who has ever eaten rabbit meat?
Where is Australia? Where is England?
Which route did Thomas and the rabbits go on their journey?
Why are there so many rabbits in Australia?
Why are there more and more rabbits?
Are there too many rabbits?

Using the globe or the world map, the students locate Australia, England and their own country. They follow the route Thomas and the rabbits took on their journey from England to Australia.

If the children already begin to think about the effects of the strong growth of the rabbit population in Australia, that's great. But the questions about the consequences and possible solutions can and should remain open at this point.

Task 2: *Number of Rabbits - Estimation and Approximation (30 minutes)*

In this task, the students should find out the number of rabbits. Therefore they look again at the last picture with a lot of rabbits.

What do you estimate how many rabbits are in the picture?
The various suggestions of the students are collected. The suggested numbers might be very different.

Can we find out more exactly how many rabbits are in the picture?
Together, the class considers what other methods can be used to determine the number of rabbits:

Can we just count all the rabbits?
That's a good idea. The students can try to count the rabbits. After a short time, this should be paused to reflect this method: There are many rabbits in the picture, that's why the counting takes so long. In addition, the rabbits are jumbled all together, so it's difficult to count them and you probably make mistakes.

How can we make it easier?
We can count the rabbits in a small area and then approximate how many there are in whole picture. Maybe the students themselves come up with this idea, otherwise the teacher should lead them there.

The individual steps of the method are briefly discussed in the class:

1. *In which area do I count the rabbits?*
2. *How many rabbits are in the chosen area?*
3. *How often does the area fit on the whole picture?*
4. *So how many rabbits are in the whole picture?*

The students then form small groups (of 2 or 3 students) in which they carry out the method. Each group receives a copy of the rabbit image. In the group they decide how big they choose the area to be counted and which form they give to the area. They can also make marks on the picture. Probably, the question of how to deal with "half" rabbits will arise. One possibility is to sum up two half rabbits to a whole one. Maybe it has to be rounded up or down.

Afterwards, the groups present their procedures and their results and compare them with each other. *Which shapes and sizes were chosen for the areas to be counted? What are the advantages and disadvantages of the different options?*

Most likely the students will have different results. *Why is that?* The rabbits are not distributed equally everywhere. So if not exactly the same area was counted, the results are different.
Are the numbers still useful? We did not find the exact number, but an approximate number. Very often that rough number gives us enough information. And very often that's also the only possible way to find out how much there is of something.

Could you count all rabbits in Australia?

Australia is very big and Australia has far too many rabbits to count all of them individually. In addition, the rabbits do not move in the picture, but in reality the rabbits hop away very quickly and can not be counted!

How could one find out how many rabbits approximately live in Australia?
This can be done similar to the way in which we calculated the number of rabbits in the picture. One can try to count the rabbits in a specific area in Australia, e.g. within one square kilometer. To be more accurate, you do not select only one area, but different ones. After all, in different areas different numbers of rabbits are living. For example, you can count how many rabbits live in one square kilometer in the city and in the countryside, in the north and south of Australia, by the sea, on the plains and in the mountains. You end up with several numbers, with which you can roughly calculate how many rabbits there are in total.

Similarly, scientists have figured out how many rabbits there approximately exist in Australia: over 200 million rabbits.

Note: It is important that the students understand that these methods and outcomes are still estimations or approximations. Although these estimations are much more accurate than an instinctive guess, they must not be understood as an exact indication. Nonetheless, it is an important and frequently used procedure whenever it is simply not possible or too expensive to find the exact number.

Task 3: *Growth of the Rabbit Population (45 minutes)*

How did the few rabbits that Thomas brought to Australia turn into over 200 million?!
This task is about the reproduction of rabbits. Therefore the students replicate the growth of a rabbit population with small rabbit pictures.
The aim of this task is that the students get to know large numbers and deal with correspondences and their representations. At the same time, they learn to reflect and evaluate different presentation options.

Note: Normally, the topic "exponential growth", which is discussed here, is adressed in higher grades; in this task the students can gain a very first insight into this topic while dealing with correspondences and their representations.

3.1 Define model assumptions

First of all, we define model assumptions for the reproduction of rabbits: *how many baby rabbits do two rabbits have in one year?*

- ➔ On average, two rabbits (one male and one female) together have about 8 descendants per year, which means one rabbit gets 4 descendants.

Together with the students, we think about what exactly that means for the number of rabbits:

- ➔ the two rabbits turn to 10 rabbits within one year (2 old + 8 young)
- ➔ one rabbit turns into 5 (1 old + 4 young)

Note: It is important to take enough time to discuss these assumptions. First of all, it should be clear to all students that these are model assumptions or averages, and naturally not all rabbits get as many descendants every year. To prepare the lessons, biological facts on the species and its reproduction may be consulted, e.g. there is information is available at Wikipedia: https://en.wikipedia.org/wiki/European_rabbit
Second, all students should understand the difference between the number of descendants per year and the total number of rabbits (including the parents).

The students may notice that rabbits die as well. If this is the case, parts of the method reflection (see below) can already be done at this point. In the following task, we pretend that no rabbits are dying - another model assumption we make.

3.2 Group work with rabbit pictures

The students form groups of 2 or 3. Each group receives a printed sheet with 35 rabbits. First, they cut out the little rabbit pictures. Their task is to use the rabbit pictures to represent the multiplication of a rabbit or the growth of the rabbit population beginning with 1 rabbit.

We start the model with one rabbit.
How many rabbits are there after one year / after 2 years / after 3 years?
How many young rabbits are there? How many rabbits are there in total?

Year	**Rabbits**
0	1
1	5
2	25
3	125

The students consider for themselves how they can best represent this with the small pictures. Most likely there will be different representations.

There aren't enough rabbit pictures for the 3rd year!
Does it work out if the small groups add up their pictures?
Maybe the students themselves come up with the idea that they don't enough pictures for the third year and that the groups can join together, otherwise this should be stimulated by the teacher.
How many groups must at least work together? At least 4 groups (4 x 35 = 140 > 125)

Note: There might be more space needed for the last part, so tables may need to be pushed together or to the edge of the room.

Possibility for differentiation: The students who already finished the task can think about how many rabbits there are after 4 years.

3.3 Gathering and reflecting the results in the classroom

After the students have placed the presentation for the 3rd year, the results are presented in class:

- *How many rabbits are there after 1, 2, 3 years?*
- *How did you come to these results?*
- *How did you represent the number of rabbits? Why?*
- *What does the chosen representation show us? What doesn't it show?*

The students should be praised for their ideas at this point. It is great that they have dealt with the question independently!

There are many different ways to visualize growth, and it is likely that the students will have very different ideas. The representations can bring different aspects into focus, e.g. the ancestry, the number of children per rabbit or the total number. Possible representations:

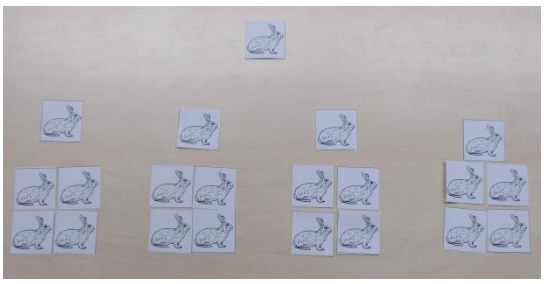

Figure 1

Figure 2

Figure 1 shows a “family tree” for the years 0-2, which puts the focus on the ancestry. Each rabbit is assigned 4 descendants. However, this illustration does not show that the first rabbit has again 4 children in the 2nd year. So you can't count how many rabbits there are in total after 2 years. Figure 2 shows an association between the year and the number of rabbits for the years 0-2. Here, the parent rabbits are counted each year.

The aim of this reflection is that the students get a sense that there are different ways to present content or the numbers. The illustrations show different aspects and do not show other aspects.

Which is the best way to represent the total number of rabbits per year?

- ➔ Correspondence: Year - Number of rabbits, as e.g. in Figure 2. This correspondence allows me to directly read the number of rabbits for the respective year.
- ➔ A good way to visualize is e.g. a bar chart, as this makes it possible to compare the numbers of rabbits per year.

3.4 Calculating further years (in the classroom)

What do you think is happening after the 3 years?
How many rabbits are there after 4, 5, 10 years?
Can we still illustrate that with the pictures? (We do not have enough pictures for that)
Is there a different way to find out the number of rabbits?

The year 4 is calculated together with the students. The PowerPoint slides with the rabbits in year 1, 2, 3 and 4 can be shown to illustrate the growth.

How does the number result from the previous one? What did we count for year 4? Can you generalize that?

The following table shows the two different calculation options, both of which should be discussed. On the PowerPoint slides the rabbits are bundled in groups of 5. This is to indicate that the calculation has something to do with the number 5.

Year	**Rabbits**

4	125*4+125 (4 young ones per rabbit plus the old ones) or 125*5 (one rabbit turns to 5 rabbits) **=625**
5	625*4+625 or 625*5 **=3125**
…	…
10	**=9 765 625**
…	…
20	**=95 367 431 640 625**

The rabbits for year 5, 6, 7, etc. are no longer fitting on the slides, so these are only shown in the form of numbers on the next slide. The children should look at the numbers on the slide and read them to get a sense of their size. Probably it's necessary to discuss the meanings of the higher digits of the numbers: thousand, ten thousand, one hundred thousand, million, billion, trillion.

3.5 Reflection of the method and the calculated numbers (with the whole class)

The students exchange ideas about the results.

What did you notice?
The numbers are very large and are getting bigger and bigger!

Do the rabbits really multiply so fast?
If they are healthy and live under good conditions they do, yes! Of course rabbits also die; we did not consider this in our model. In addition, a rabbit, that is for example sick or finds little food, gets less or no children. So there are several factors that limit the growth of the rabbit population.

For what reasons can it happen that the number of rabbits grows more slowly than in our model or maybe not at all?
The students collect potential factors: food availability; natural enemies like foxes; natural, age-related death; diseases; rabbits are run over; people hunt rabbits to eat them or to use their fur…

→ There is usually a balance in nature, which ensures that there are not too many of the different living beings, so that all have space, enough food, etc.

What about the rabbits in Australia?
The rabbits that came to Australia with Thomas were doing very well. They had very much to eat, plenty of space and no or hardly any enemies. That's why they multiplied so fast.

Task 4: *Preparation of the Role Playing Game (30 minutes)*

To prepare for the role playing game (task 5), the students explore the implications of the growth of the rabbit population in Australia, their consequences and what has been done to balance the ecosystem. Therefore the students receive the text "Rabbits in Australia", which they should read carefully. Afterwards, they can talk with their neighbour about the text.

Text-related questions are discussed in class. But the content should not be discussed or evaluated at this point, as this is part of the role playing game.

If enough time is available, alternatively an internet research can be carried out. Therefore the students themselves find out what the consequences of the high growth of the rabbit population in Australia are and what measures have been taken. The results can then be collected in class.

Task 5: *Role Playing Game (1 hour)*

In this last activity, the students independently engage with the interests of different groups in Australia. Together they try to find compromises and develop solutions.

First of all, the teacher explains the idea / the goal of the role playing game: In Australia there live different people and animals with very different needs, opinions and interests. In the game, different people and animals meet, talk about their interests and try to agree on something, that ideally makes everybody happy!

5.1 Get to know the roles

First, the students form pairs. Each pair is assigned a role they will later represent in the discussion. In total, there are four roles (rabbit, farmer, teacher, observation) plus an optional fifth role (animal rights activist). The animal rights activist has similar interests as the rabbit; this role can be used for differentiation in the class. Later, 4 (or 5) pairs with different roles will form a discussion group.

Each pair receives a role description, which they read carefully. Afterwards, the two students talk about their role. They try to empathize with the role and gather arguments for it.

What interests does your role have?
- What is important for your role? Why?
- What does your role need (to survive)?

How can you represent the interests of your role?
- What are the arguments for your position?
- What can / must your role do to feel better?
- What can / must others do to help your role feel better?

The groups with the role *observation* also read their card carefully. Their job is to watch the discussion. During the preparation time, they talk in pairs about their task and their observation questions:

What exactly is the task of the observation role?
What is meant by the individual observation questions?
What do you expect from the discussion?
Do you think there will be a solution?

5.2 Discussion

First, the process is described to the students and it is announced that the role playing game starts and ends with a certain sound (show). In between, the role playing game can also be interrupted and restarted with the sound. The pupils should slip into their role with the first sound. With the next sound they (briefly) leave their role.

4 (or 5) pairs with different roles come together in a discussion group. The discussion will take place in different phases, which are guided by the teacher (to announce the next phase the role-play can be interrupted with the sound).

Phase 1: Exchanging arguments
One after the other, the different roles introduce themselves and tell what is important to them.
Who are you and what do you want?

Phase 2: Arguing and negotiating
Now, the students know their own interests and those of the other roles. They think together about what they can do. Of course, they continue to stand up for their own interests and exchange their arguments.
What can you do? What do you want to do?
Try to convince the other roles of your interests!

Phase 3: Searching a solution / compromise
Now the students try to find a solution that everyone agrees with.
What possible solutions are there?
Is there a solution that makes everyone happy?
Can you agree on a common solution? Can you find a compromise?
If so, what is your solution?
If not, why didn't you find a solution?

In due course, the discussion ends with a final sound.

5.3 Reflection

The groups now exchange their experiences and present their solutions or compromises to each other (if they found one).
Therefore, the two observers of the group present what they noticed during the discussion, starting from the questions on their role card:

1. Which views do the different roles have?
2. What do they want to achieve?
3. Which solution proposals are made?
4. Was a common solution found? Which one? Why?
5. Did everyone agree with the solution?
6. If no common solution has been found: why not?
7. If a common solution has been found: Do you think the solution is realistic?

Afterwards, the other group members can add their own experiences.
Did you find it easy / difficult to put yourself in the role?
How did you feel during the role playing game?
Were the interests of your role taken seriously?

All presented discussion results and all ideas for solutions should be appreciated. It is great that the students have independently thought about this conflict and tried to unite their interests.
At the same time, it should be considered that some solution ideas might not be realistic; e.g. *can rabbits decide to have just one child each?* Nevertheless, all solutions are equally valuable in the role playing game (in which the rabbits can speak as well)!

Extending the learning

To deepen the mathematical understanding, the calculations and illustrations can be repeated with other (invasive) species.

In addition, the growth of the world's human population can be addressed (from a mathematical and a social perspective). However, a comparison between rabbit growth and population growth should be carried out critically and sensitively.