

# Conceptual and methodological principles for the curriculum materials, eTwinning and teacher education modules

## Introduction

During the first transnational meeting of the PiCaM project held in Sheffield, UK in November 2017, the project partners created the first version of a giant matrix which sought to explore the connections between mathematics and global citizenship. In order to promote creative and divergent thinking, partners initially created column and row headers, embracing both mathematics and global citizenship curriculum content and competences. The intersections of rows and columns were then populated with ideas for learning opportunities. The matrix was revised during the weeks that followed and a typed up version of the initial matrix can be found in Figure 1.



## The mathematics

As part of establishing a shared understanding of the nature of mathematics and its teaching and learning, the project considered the pedagogical principles which should underlie the teaching; the 'big ideas' in mathematics that children might be expected to meet; and the 'habits of mind' that PiCaM should seek to inculcate in learners<sup>1</sup>.

The 'habits of mind' are mathematical competences required for successful, engaged and critical learning of the subject as reflected in the matrix:

- looking for patterns and connections
- asking yourself questions
- being organised and systematic
- being resilient and flexible
- conjecturing and checking things out
- visualising, imagining and using intuition
- using embodied and multi-sensory approaches
- using representation and symbolism
- modelling and dealing with uncertainty
- using analogies
- using argumentation and reasoning.

<sup>1</sup> These points are elaborated in Povey, H. (2017) *Engaging (with) Mathematics and Learning to Teach* Münster: WTM. They are given physical form in the mathematical hands-on exhibits at *Mathematikum* in Giessen in Germany.

Global									
Coming back to our bodies & senses		Architectural constructions with dowling and rubber bands	Dance, movement, how people count - different patterns, rhythms	1. Body numbers. 2. Multiplication factors thru' ensemble	What is control? What does it help you to do? And not do?	Measuring emotions/happiness/...	Performance/dance & mathematics		Is 'uncertainty' helpful? When do you need to order chaos?
Values, Perceptions (myths, realities, manipulations)	Map projections		Types of functions (linear, exponential)	Does Google know me?	Artificial Intelligence (chances, risks)	Can we trust numbers?			1. Empowerment. 2. Right, wrong, inbetween?
Inequality, inequity, unfairness (& peace)	Global distribution of goods		Global transport (oil, food ...)			Reading & tweaking statistics	Maps / Representations of global inequality	Voting Systems	
Empowerment, freedom, autonomy, agency (& citizenship)	1. Primitives' & space -> self improvisations, (music, art, dance) 2. Disability & space					System Thinking activities	Political (mis) representations	Project exploring maths zero/ infinity etc.	
Sustainability			Virtual water, CO2 footprint, slavery footprint		How communities self-organise to survive (e.g. % income to common goods)	Prognosis / prediction	Virtual reality	P4C enquiry on Climate change/chaos	New shapes in 4-D or n-D New Worlds!
Valuing cultures & histories	1. What is beautiful? 2. Sharing words for/in maths.			1. Use music from diff cultures to explore math sequences/patterns. 2. Number systems & algorithms	The story of algorithm and its presence		1. Art & Mathematics. 2. Beads, mandalas & religious symbols	Design rules in arts/crafts	
Interdependence, Globalisation & Pluralism	Islamic tiles and geometry. Borders & boundaries			Money systems (exch rates) different calendar systems, datelines		1. Numerical (?) measures of the world. 2. Waste / consumption globally			
Rights? SDGs?									
<b>MATHEMATICS</b>	SHAPES, FIGURES, AREA, VOLUME, PATTERN	STRUCTURE, ORDER, REGULARITY, RULES	FUNCTIONS (MOVEMENT, CHANGE)	NUMBERS, POSSIBILITY OF CALCULATIONS, COORDINATES, NUMBER SYSTEMS	ALGORITHM, CONTROL, DISCIPLINE, ERROR - CORRECTION	RANDOMNESS, UNPREDICTABILITY, STATISTICS, CHANCE, DATA	REPRESENTATIONS	RATIONALITY, LOGIC, FANTASY, IMAGINATION, IMAGERY, INTUITION, REASONING, ARGUMENTATION	FREEDOM, NECESSITY, TRUE / FALSE, TOPOLOGIES, CHAOS, ENIGMA, DISCONTINUITY

Figure 1: Initial PiCaM matrix

PiCaM's understanding of 'big ideas' is influenced by the concepts behind Philosophy for Children. Children are fascinated by big ideas (the 'wow' factor) and can work with concepts that stretch beyond the school syllabus, such as infinity and discontinuity, control/chaos, certainty/enigma and topological ideas of space. They can experience structure and pattern and the meaning of equivalence. They can work with change and notice what remains invariant and can learn to appreciate the difference between the arbitrary and the necessary. The importance of these last two experiences extends far beyond mathematics.

Another very important influence on the thinking about mathematical activities was the importance of learning opportunities which help us in 'coming back to our bodies and our senses', challenging the objectivity and neutrality of mathematics and the tendency for its use to privilege objects over people. This might be the incorporation of sensory and movement approaches that immerse mathematics in 'the drama of life'.<sup>2</sup>

A number of pedagogical principles will also underpin PiCaM. Partners recognise the need for a pedagogy which is based on building a learning community in which the attainment of the group is valorised rather than the attainment of the individual and interpersonal connections in the classroom are based on relational equity. The classroom should be a dialogic space in which all learners develop their own epistemological authority and should allow both time and space for ideas to develop, to be revisited and to be nurtured. Attention should always be paid to the historical and cultural aspects of

<sup>2</sup> Such approaches like 'Street Mathematics' are described in Chronaki, A (2017) Assembling Mathlife Chronotopes *Proceedings of the 9th International Conference of Mathematics Education and Society* 2, 427.

mathematics and the importance recognised of the artefacts through which the curriculum is mediated and within which it is experienced.

As noted above, Philosophy for Children has influenced the thinking of the PiCaM partners. The teachers' guide states: *Philosophy for Children is a way of learning and teaching, where children become more thoughtful and reflective and go beyond information to seek understanding. Children learn how to participate in meaningful discussions, where their ideas and those of others are valued and listened to.*<sup>3</sup> PiCaM partners argue that these dispositions and skills should underlie all learning including the learning of mathematics within a pluralist, democratic society.

The Philosophy for Children guide suggests prompts for teachers to stimulate thinking and these will inform the curriculum materials of the project. These include, for example,

- What don't we understand here?
- What questions do we have about this?
- Does anyone have any alternative suggestions or explanations?
- What reasons are there for doing that?
- Can anyone think of an example of this?
- Can someone think of a counterexample?
- Is anyone able to build on that idea?
- What assumptions lie behind this?
- What makes that an example of X?
- Does that conclusion follow?

The partners also brought together the top-level areas of the syllabus structures in mathematics for the target age group (9-12 years) across the five PiCaM partner countries and identified some common themes and some common core competences. These are collated in Figure 2.

---

<sup>3</sup> Oxfam, Philosophy for Children Teachers Guide <https://www.oxfam.org.uk/education/resources/philosophy-for-children>

- **Mathematics Syllabus Structures across the PiCaM countries for 9 – 12 year olds (approx)**

	Numbers & Place Value	Number Operations	Fractions & Decimals	Measurement	Geometry (Space & Shape)	Functions & Sequences (& Successions)	Algebra (& Equations)	Data, Stats (& Randomness)	Analogies
Germany	✓	✓	✓	✓	✓	✓		✓	
Greece	✓	✓		✓	✓		✓	✓	✓
Portugal	✓	✓		✓	✓	✓	✓	✓	
England	✓	✓	✓	✓	✓		✓	✓	
Romania	✓	✓	✓	✓	✓		✓	✓	✓

- **Mathematics competences across the PiCaM countries for 9 – 12 year olds (approx)**

	Argumentation	Problem Solving	Modelling	Using Representations	Using Symbolism (Mathematical Language)	Communication & Explanation	Resilience	Visualising	Being Systematic	Guessing then Checking
Germany	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Greece	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Portugal										
England	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Romania	✓	✓	✓	✓	✓	✓		✓		

• **Figure 2: Mathematics syllabus structures and competences across the PiCaM countries**

## Global citizenship

Since PiCaM operates across five countries and aims to work with children and teachers from diverse communities, the notion of ‘global’ rather than national citizenship is being adopted. The term has been used by the NGO Oxfam since 1996 (with revised guidance issued in 2015)<sup>4</sup> but terms like ‘World Citizenship’ or ‘Cosmopolitanism’ date back much further. Most recently the OECD has recognised the importance of ‘Global Competences’ in the 2018 PISA assessments. The most recent guidance<sup>5</sup>, which relates well to PiCaM’s aims, ties these competences in with the UN’s Sustainable Development Goals (see Annex 1). They formed the basis of the global citizenship topics and competences in the PiCaM matrix.

However, following discussions with PiCaM partners, it was decided that the high profile OECD components of global competence were valuable, even necessary, but not sufficient for our purposes. This is because, as in most areas of human endeavour, global citizenship can be described from broadly conservative, liberal or radical perspectives, depending on the purposes and interests of its proponents. The OECD framework seems to be a little weak on radical post-colonial and indigenous perspectives. We have therefore also drawn on the Earth CARE (i) Framework<sup>6</sup>. This proposes a vision of deep transformational learning processes that combine:

- practical doing (together)
- the building of trust (in one another)
- deepening analyses (of self, systems, and social and ecological complexity)
- dismantling walls (between peoples, knowledges, and cultures).

In this vision, intellectual engagements, the arts, ethics, cosmo-visions, the environment and embodied practices are all understood as important conduits for learning.

Its six complementary approaches to justice are summarised below.

- *ecological justice*: focusing on social-ecological integration, food security, soil regeneration, and ‘living well’ as opposed to ‘living better’ (sumak kawsay);
- *cognitive justice*: identifying the limits of current paradigms and creating new ‘dispositions of engagement’ with mainstream and alternative knowledge systems and technologies;
- *affective justice*: recognizing our collective need for healing from historical and inter-generational trauma, prioritizing collective well-being;
- *relational justice*: dismantling divisions caused by inherited social, cultural, economic and epistemological hierarchies that hinder symmetrical relationships;
- *economic justice*: analyzing and acting upon the systemic reproduction of inequalities through unjust systems of trade, governance and value production, while identifying viable possibilities for economic dignity; and,
- *intergenerational justice*: securing relationships and forms of organization that can uphold the health and wellbeing of present and future generations.

Elements from this framework have been woven in between the OECD derived topics in the revised PiCaM matrix which appears overleaf as Figure 3. For example ‘alternative knowledge systems’ is drawn from *cognitive justice* and appears next to the ‘culture and intercultural relations’ domain from the

---

<sup>4</sup> Oxfam (2015) [Education for global citizenship a guide for schools](#) Oxford: Oxfam

<sup>5</sup> OECD (2018) [Preparing our youth for an inclusive and sustainable world, the OECD global competence framework](#) Paris: OECD

<sup>6</sup> The Earth CARE (i) Community (2017) [The Earth CARE \(i\) Framework](#) Vancouver: EarthCare(i)

OECD. In a similar way 'legacy of colonialism', derived from *affective* and *relational justice*, appears next to the 'socio-economic development and interdependence' OECD domain. 'Buen vivir' or 'sumak kawsay' (in the Quechua language) goes beyond many western notions of environmental sustainability as it regards humans and the natural world as being so interconnected as to be part of one greater metabolism.

### **The PiCaM conceptual and methodological matrix**

The ideas for learning opportunities or activities from the initial version of the PiCaM matrix prepared at the transnational meeting have been transposed, with the new row and column headings, into the revised matrix as in Figure 3. The competences have been listed separately from the topics and have been numbered, or lettered, so that they can be referenced in each learning opportunity. The additional activities are shown in purple. The matrix is available as an Excel spreadsheet on the PiCaM website.

The activities are not intended as a blueprint for the next stages in PiCaM - the curriculum materials, the e-twinning and the teacher education modules - but to exemplify meanings in the matrix. Some of the activities will be worked on further, some will not and some new ideas are also likely to emerge.

However, the principles established by the partners and presented in this document will offer significant guidance in this next PiCaM developmental stage.

Project in Citizenship and Mathematics (PiCaM)

MATHEMATICS COMPETENCES								
	1) LOOKING FOR PATTERNS & CONNECTION S							
	2) ASKING YOURSELF QUESTIONS							
	3) BEING SYSTEMATIC							
	4) BEING RESILIENT & FLEXIBLE							
	5) CONJECTURING & CHECKING THINGS OUT							
	6) VISUALISING, IMAGINING & USING INTUITION							
	7) USING EMBODIED & MULTI-SENSORY APPROACHES							
	8) USING REPRESENTATIONS & SYMBOLISM							
	9) MODELLING & DEALING WITH UNCERTAINTY							
<b>Global Citizenship Topics</b>	10) USING ANALOGIES							
	11) USING ARGUMENTATION & REASONING							
<b>Inequalities &amp; Hierarchies</b>			<i>Graphical representations</i>		<i>Distribution of goods</i>	<i>Global transport: eg oil, food</i>	<i>Inequality &amp; wellbeing</i>	<i>Critical interpretation of statistics</i>
<b>Institutions Conflicts &amp; Human Rights</b>					<i>Maps as spatial representations</i>	<i>Slavery footprint</i>	<i>Empowerment; Right, wrong, ?</i>	<i>Voting systems</i>
<b>Buen ViVir (social ecological integration)</b>				<i>Population density</i>	<i>Indigenous people and space</i>	<i>Communities self-organising (eg. % tax)</i>		<i>System thinking activities; Predictions</i>
<b>Environmental Sustainability</b>				<i>CO<sub>2</sub> &amp; temperature</i>		<i>Virtual H<sub>2</sub>O; CO<sub>2</sub> footprint</i>		<i>Consumption &amp; waste</i>
<b>Legacy of Colonialism</b>			<i>Land distribution</i>		<i>Borders &amp; boundaries</i>	<i>Trade rules</i>		<i>Flow of goods &amp; money</i>
<b>Socio-economic Development &amp; Inter-dependence</b>	<i>Money, exchange rates, calendars datelines</i>				<i>Disability &amp; space</i>	<i>Artificial intelligence; Types of function: (linear, exponential)</i>		<i>Measuring emotions &amp; happiness</i>
<b>Alternative Knowledge Systems</b>	<i>Body numbers</i>	<i>Multiplication factors by ensemble</i>		<i>Body measures</i>	<i>Maps and map projections</i>	<i>What is control? Help or hinder?</i>	<i>Is uncertainty helpful?</i>	<i>Kinaesthetic representations of data</i>
<b>Culture &amp; Intercultural Relations</b>	<i>Different number systems</i>	<i>Does Google know me?</i>	<i>Decimal systems in different cultures</i>	<i>Measures of the world</i>	<i>Islamic tessellation &amp; 3D models</i>	<i>Sequences in music and dance of different cultures</i>	<i>The story of algebra</i>	
<b>Mathematics Curriculum Topics</b>	<b>Numbers &amp; Place Value</b>	<b>Number Operations</b>	<b>Fractions &amp; Decimals</b>	<b>Measurement</b>	<b>Geometry (Space &amp; Shape)</b>	<b>Functions, Sequences &amp; Algorithms</b>	<b>Algebra &amp; Equations</b>	<b>Data, Statistics &amp; Randomness</b>
<b>GLOBAL CITIZENSHIP COMPETENCES</b>								
A) CAPACITY TO EXAMINE GLOBAL ISSUES								
B) APPRECIATE DIFFERENT PERSPECTIVES & WORLDVIEWS								
C) POSITIVE INTERACTIONS WITH PEOPLE WHO ARE DIFFERENT								
D) TAKE CONSTRUCTIVE ACTION FOR SD & SOCIAL WELL-BEING								
E) ANALYTICAL & CRITICAL THINKING SKILLS								
F) COMMUNICATION & CO-OPERATION SKILLS								
G) CONFLICT RESOLUTION SKILLS								

Figure 3: Revised PiCaM matrix

## Annex 1

The OECD Global Competences are divided into four dimensions:

- Dimension 1: Examine issues of local, global and cultural significance.
- Dimension 2: Understand and appreciate the perspectives and world views of others.
- Dimension 3: Engage in open, appropriate and effective interactions across cultures.
- Dimension 4: Take action for collective well-being and sustainable development.
- 

The content (for the scenarios used in the assessments) is divided into four domains and associated subdomains:

### *Content Domain 1: Culture and intercultural relations*

- Subdomain 1.1: Identity formation in multicultural societies
- Subdomain 1.2: Cultural expressions and cultural exchanges
- Subdomain 1.3: Intercultural communication
- Subdomain 1.4: Perspective taking, stereotypes, discrimination and intolerance

### *Content Domain 2: Socio-economic development and interdependence*

- Subdomain 2.1: Economic interactions and interdependence
- Subdomain 2.2: Human capital, development and inequality

### *Content Domain 3: Environmental sustainability*

- Subdomain 3.1: Natural resources and environmental risks
- Subdomain 3.2: Policies, practices and behaviours for environmental sustainability

### *Content Domain 4: Institutions, conflicts and human rights*

- Subdomain 4.1: Prevention of conflicts and hate crimes
- Subdomain 4.2: Universal human rights and local traditions
- Subdomain 4.3: Political participation and global engagement

These components are derived from the auditing of 101 existing conceptual global, intercultural and civic competence schemes by the Council of Europe (2016) [Competences for Democratic Culture: Living Together as Equals in Culturally Diverse Democratic Societies](#), Council of Europe, Strasbourg.